

Example of removing left-recursion in a grammar

Left-recursive grammar:

$$\underline{e} \rightarrow \underline{e} + \underline{t}$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{t} \rightarrow \underline{t} * \underline{f}$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

Rule 1

$$\underline{n} \rightarrow \underline{n} \alpha$$



$$\underline{n}' \rightarrow \alpha \underline{n}'$$

$$\underline{e} \rightarrow \underline{e} + \underline{t}$$



$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{t} \rightarrow \underline{t} * \underline{f}$$

$$\underline{t} \rightarrow \underline{t} * \underline{f}$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

$$\underline{f} \rightarrow y$$

Rule 1

$$\underline{n} \rightarrow \underline{n} \alpha$$



$$\underline{n}' \rightarrow \alpha \underline{n}'$$

$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{t} \rightarrow \underline{t} * \underline{f}$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$



$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{t}' \rightarrow * \underline{f} \underline{t}'$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

Rule 2

$$\underline{n} \rightarrow \alpha \quad \Rightarrow \quad \underline{n} \rightarrow \alpha \underline{n}'$$

where α does not begin with \underline{n}

$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t}$$

$$\underline{t}' \rightarrow * \underline{f} \underline{t}'$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$



$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t} \underline{e}'$$

$$\underline{t}' \rightarrow * \underline{f} \underline{t}'$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

Rule 2

$$\underline{n} \rightarrow \alpha \quad \Rightarrow \quad \underline{n} \rightarrow \alpha \underline{n}'$$

where α does not begin with \underline{n}

$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t} \underline{e}'$$

$$\underline{t}' \rightarrow * \underline{f} \underline{t}'$$

$$\underline{t} \rightarrow \underline{f}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$



$$\underline{e}' \rightarrow + \underline{t} \underline{e}'$$

$$\underline{e} \rightarrow \underline{t} \underline{e}'$$

$$\underline{t}' \rightarrow * \underline{f} \underline{t}'$$

$$\underline{t} \rightarrow \underline{f} \underline{t}'$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

Rule 3

Introduce new
production

$$\underline{n'} \rightarrow \varepsilon$$

$$\underline{e'} \rightarrow + \underline{t} \underline{e'}$$

$$\underline{e'} \rightarrow \varepsilon$$

$$\underline{e} \rightarrow \underline{t} \underline{e'}$$

$$\underline{t'} \rightarrow * \underline{f} \underline{t'}$$

$$\underline{t} \rightarrow \underline{f} \underline{t'}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$

Rule 3

Introduce new
production

$$\underline{n'} \rightarrow \varepsilon$$

$$\underline{e'} \rightarrow + \underline{t} \underline{e'}$$

$$\underline{e'} \rightarrow \varepsilon$$

$$\underline{e} \rightarrow \underline{t} \underline{e'}$$

$$\underline{t'} \rightarrow * \underline{f} \underline{t'}$$

$$\underline{t'} \rightarrow \varepsilon$$

$$\underline{t} \rightarrow \underline{f} \underline{t'}$$

$$\underline{f} \rightarrow (\underline{e})$$

$$\underline{f} \rightarrow x$$

$$\underline{f} \rightarrow y$$